Symmetric eigenvalue problems of the form \(Ax = \lambda Bx\) can be solved uniquely if \(A\) and \(B\) are symmetric and \(B\) is positive definite, as long as appropriate scaling conventions are understood.

From the SimFit main menu choose [Statistics] then [Numerical analysis] and open the procedure to solve symmetric eigenvalue problems. From this control you are given the options to solve any of the following three problems.

\[
\begin{align*}
Ax & = \lambda Bx \\
ABx & = \lambda x \\
BAx & = \lambda x
\end{align*}
\]

The SimFit default test files are `matrix.tf4` containing matrix \(A\), and `matrix.tf3` containing matrix \(B\) as now displayed.

\[
\begin{array}{cccc}
0.24 & 0.39 & 0.42 & -0.16 \\
0.39 & -0.11 & 0.79 & 0.63 \\
0.42 & 0.79 & -0.25 & 0.48 \\
-0.16 & 0.63 & 0.48 & -0.03 \\
\end{array}
\]

\[
\begin{array}{cccc}
4.16 & -3.12 & 0.56 & -0.10 \\
-3.12 & 5.03 & -0.83 & 1.09 \\
0.56 & -0.83 & 0.76 & 0.34 \\
-0.10 & 1.09 & 0.34 & 1.18 \\
\end{array}
\]

The results from analyzing the standard problem \(Ax = \lambda Bx\) are then as follows.

\[
\begin{array}{cccc}
\text{Eigenvalues...Case: } Ax = \lambda Bx \\
-2.2254476E+00 \\
-4.5475588E-01 \\
1.0007648E-01 \\
1.1270387E+00 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Eigenvectors by column ...Case: } Ax = \lambda Bx \\
-6.9005765E-02 & 3.0795498E-01 & -4.4694499E-01 & -5.5278790E-01 \\
-5.7401486E-01 & 5.3285741E-01 & -3.7084023E-02 & -6.7660179E-01 \\
-1.5427579E+00 & -3.4964452E-01 & 5.0476980E-02 & -9.2759211E-01 \\
1.4004070E+00 & -6.2110938E-01 & 4.7425180E-01 & 2.5095480E-01 \\
\end{array}
\]

It should be noted that the eigenvectors are the columns of a matrix \(X\) that is normalized so that

\[
X^TXB = I, \text{ for } Ax = \lambda Bx, \text{ and } ABx = \lambda x,
\]

\[
X^T B^{-1}X = I, \text{ for } BAx = \lambda x.
\]

where \(I\) is the identity matrix.

Warnings will be issued if there is a clash of dimensions, or \(A\) and \(B\) are not symmetric, or \(B\) is not positive definite.