When a time series has been analyzed for non-seasonal and seasonal differencing it is possible to fit an autoregressive integrated moving average model (ARIMA) and obtain forecasts for future values.

From the main SimFit menu choose [Statistics], [Time series], then the [ARIMA with forecasts] option and fit the test file time tf1 using the default settings to obtain the following results, and plots.

**ARIMA with forecasts**

Current data title is:
Test file times.tf1: time series data (J06SBF)
Original dimension (NX) 100
After differencing (NXD) 99
Non-seasonal order (ND) 1
Seasonal order (NDS) 0
Seasonality (NS) 0
Number of forecasts (NF) 3
Number of parameters (NP) 1
Number of iterations (ITC) 2
Sum of squares (SSQ) 19.9092

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std. err.</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi(1)$</td>
<td>0.60081</td>
<td>0.08215</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>$C(0)$</td>
<td>0.42960</td>
<td>0.11239</td>
<td>Constant term</td>
</tr>
<tr>
<td>prediction(1)</td>
<td>43.9351</td>
<td>0.45305</td>
<td>Forecast</td>
</tr>
<tr>
<td>prediction(2)</td>
<td>44.2082</td>
<td>0.85512</td>
<td>Forecast</td>
</tr>
<tr>
<td>prediction(3)</td>
<td>44.5437</td>
<td>1.23335</td>
<td>Forecast</td>
</tr>
</tbody>
</table>

**ARIMA forecasts with 95% Confidence Limits**

![Graph showing ARIMA forecasts with 95% Confidence Limits](image-url)
Theory

It must be stressed that fitting an ARIMA model is a very specialized iterative technique that does not yield unique solutions. So, before using this procedure, you must have a definite idea, by using the auto-correlation and partial auto-correlation options or by knowing the special features of the data, exactly what differencing scheme to adopt and which parameters to fit. Users can select the way that starting estimates are estimated, they can monitor the optimization, and they can alter the tolerances controlling the convergence, but only expert users should alter the default settings.

It is assumed that the time series data \( x_1, x_2, \ldots, x_n \) follow an ARIMA model so that a differenced series given by

\[
\nabla^d \nabla^D_s x_i = c
\]

can be fitted, where \( c \) is a constant, \( d \) is the order of non-seasonal differencing, \( D \) is the order of seasonal differencing and \( s \) is the seasonality. The method estimates the expected value \( c \) of the differenced series in terms of an uncorrelated series \( a_t \) and an intermediate series \( e_t \) using parameters \( \phi, \theta, \Phi, \Theta \) as follows. The seasonal structure is described by

\[
w_t = \Phi_1 w_{t-s} + \Phi_2 w_{t-2s} + \cdots + \Phi_p w_{t-ps} + e_t - \Theta_1 e_{t-s} - \Theta_2 e_{t-2s} - \cdots - \Theta_Q e_{t-Qs}
\]

while the non-seasonal structure is assumed to be

\[
e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \cdots + \phi_p e_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q}.
\]

The model parameters \( \phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q \) and \( \Phi_1, \Phi_2, \ldots, \Phi_p, \Theta_1, \Theta_2, \ldots, \Theta_Q \) are estimated by non-linear optimization, the success of which is heavily dependent on choosing an appropriate differencing scheme, starting estimates and convergence criteria. After fitting an ARIMA model, forecasts can be estimated along with 95% confidence limits.